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Role of parallel electric field on the growth of ion cyclotron instability in magnetospheric plasma

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The existence of a parallel electric field has created considerable interest in VLF wave propagation and emission studies. The acoustic wave amplification at microwave frequencies in the presence of both magnetostatic field and electrostatic field has been examined by Eers and Brueck (1968). Hsieh (1967, 1968a, b) has investigated the effect of longitudinal as well as transverse electric field on the propagation of whistler mode waves in the ionosphere by different technique. The cyclotron resonance between circularly polarized electromagnetic waves and energetic charged particles is one of the dominant energy exchange processes in the magnetosphere. For anisotropic plasma distribution, the energy transfer leads to the amplification of signals over a broad band of frequencies. Cornwall and Schulz (1971) discussed the electromagnetic ion-cyclotron instability in the presence of cold plasma and gave analytical expressions for the growth rates as a function of wave frequency ω. Further ion cyclotron electromagnetic instability in the presence of warm and cold plasma was studied in detail by Cuperman *et al* (1975).

In present communication, the effect of parallel electrostatic field on the amplification of ion cyclotron electromagnetic wave propagating along the magnetic field in an anisotropic Maxwellian weakly ionized magnetoplasma has been studied. The expression for the growth rate for temperature anisotropy and electric field of the order of few millivolts per metre in collision-less Maxwellian plasma has been derived which becomes similar to the expression of Kennel and Petschek (1966) in absence of applied electric field. The growth rates by using plasma parameters similar to those observed in the equatorial magnetosphere at 6.6 R_E have been computed and results have been discussed.

The applied electric field E_0 parallel to magnetic field B_0 has the effect to modify the particles thermal velocity in that direction from α_s to the complex value α_{s0} which is equivalent to say that the temperature T_s in the direction of magnetic field modifies to the complex temperature $T_{\parallel o}$ (Pines and Schrieffer 1961, Bers and Brueck 1968) as

$$T_{\parallel o}=T_{\parallel}(1+iqE_0/kKT_{\parallel}) \tag{1}$$

where q is the charge of the particles, k is the wave number and K is the Boltzmann constant. After including the effect of parallel electric field, the dispersion relation is written in terms of the Fried and Conte Plasma dispersion function as

$$\omega^2=k^2c^2-\omega_p^2\left[\frac{\omega}{k\alpha_{\parallel o}}Z(\xi)+\bar{A}(1+\xi Z(\xi))\right] \tag{2}$$

where

$$A=\left(\frac{\alpha_\perp}{\alpha_{\parallel o}}\right)^2-1=\frac{T_\perp}{T_{\parallel o}}-1$$

$$Z(\xi)=\frac{1}{\pi^{\frac{1}{2}}}\int_{-\infty}^{\infty}\frac{e^{-t^2}}{(t-\xi)}\,dt$$

$$\xi=\frac{(W-WH)}{k\alpha_{\parallel o}}.$$

This equation for the imaginary part of the frequency, determines the growth or damping of the wave for a given set of plasma parameters. In above eqs, ω is the wave frequency, ω_H is the ion cyclotron frequency and ω_p stands for plasma frequency.

In order to obtain the expression for the growth rate γ, the dispersion relation (2) has been solved by using the asymptotic expansion of dispersion function $Z(\xi)$ with the following assumptions

$$|\xi|\gg 1,\ \text{Imaginary}\ (\xi)\ll \text{Real}\ (\xi)$$

and $c^2k^2/\omega^2\gg 1$ which can be expressed as

$$Z(\xi)=i\pi^{\frac{1}{2}}\exp(-\xi^2)-\frac{1}{\xi}\left(1+\frac{1}{2\xi^2}\right) \tag{3}$$

and substituting into dispersion relation (2) for $c^2k^2/\omega^2\gg 1$, the expression reduces to

$$\left[\frac{c^2k^2}{\omega_p^2}+\frac{X}{X-1}+\frac{XY}{2\phi}-Y(A+1)\left(\frac{X-1}{2\phi}\right)\left(1-\frac{(X-1)^2}{\phi^2}\right)\right]$$

$$-i\left[\frac{\pi^{\frac{1}{2}}}{\phi}\exp\left[-\frac{(X-1)^2}{\phi^2}\right]\left[\left(\frac{Y^2}{2}+A\right)(X-1)-1\right]+2Y\frac{X}{1-X}\right]=0 \tag{4}$$

where

$$X=\frac{\omega}{\omega_H},\quad Y=q\frac{E_0}{kKT_z},\quad \phi=\frac{k\alpha_z}{\omega_H}.$$

Eq. (4) can be written in the form

$$D=R_e(D)+iI_m(D)=0 \tag{5}$$

by using the basic definition of growth rate γ as

$$\gamma=\frac{-I_m(D)}{\omega_H\partial/\partial\omega R_e(D)} \tag{6}$$

The real part of the dispersion relation (4) for negligibly small value of Y gives rise to

$$\frac{X}{1-X}=\frac{\phi^2}{\beta} \tag{7}$$

where $\beta=\mu_0 n_0 KT_z/B_0^2$, μ_0 is the permeability of free space. The expression for the growth rate, γ is found to be

$$\gamma=\frac{\frac{\pi^{\frac{1}{2}}}{\phi}\exp\left[\frac{1}{\phi^2(1+\phi^2/\beta)}\right]\left[(A-\phi^2/\beta)+\frac{Y^2}{2}\right]+2Y\phi^2/(1+\phi^2/\beta)}{\frac{Y}{2\phi^3}\left[1+A\left[1+\phi^2\left(1+\frac{\phi^2}{\beta}\right)\right]\right]+\left(1+\frac{\phi^2}{\beta}\right)^3} \tag{8}$$

when $E_0=0$ the expression for growth rate γ reduces to that of Kennel and Petschek (1966)

$$\gamma=\frac{\pi^{\frac{1}{2}}}{\phi}\left(A-\frac{X}{1-X}\right)(1-X)^3\exp\left[\frac{(1-X)^2}{\phi^2}\right] \tag{9}$$

The plasma parameters used in this paper have been chosen from the megnetospheric conditions at geostatic equatorial altitude (Williams 1970) and the value of the electric field have been taken from McCormac (1971) as $\beta=0.5$ and $E_0\leqslant 20$ mV/m at 6.6 R_E where R_E is the earth radii. In Figure 1, variation of the growth rate γ $V_z\phi(=k\alpha_z/\omega_H)$ has been shown for various values of parallel electrostatic fields E_0 and temperature anisotropy $A=5$ for fully ionized hydrogen plasma at $L=6.6$ R_E. It is noted that the effect of the electric field on growth rate is to enhance the range of $\phi\left(=\frac{k\alpha_z}{\omega_H}\right)$ values for a given temperature anisotropy ($A(=T_\perp/T_z-1)$ to lower limits or towards lower wavelengths. The variation in growth rate with respect to temperature anisotropy for various values of electric field at

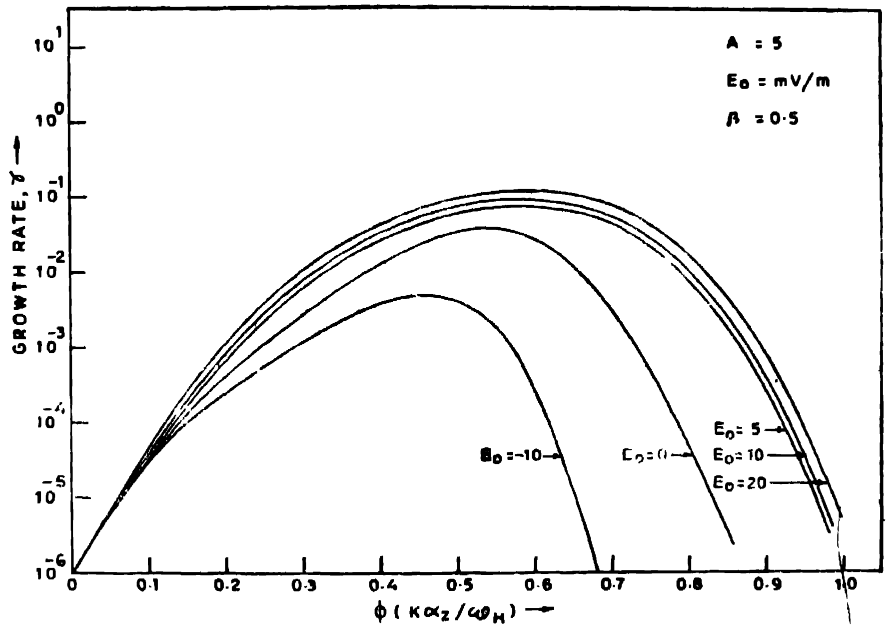

Figure 1. The variation of growth rate γ with $\phi\left(\frac{k\alpha_z}{\omega_H}\right)$ for different values of E_0, $A\left(\frac{T_\perp}{T_\parallel}-1\right)=5$ at $L=6.6\ R_E$.

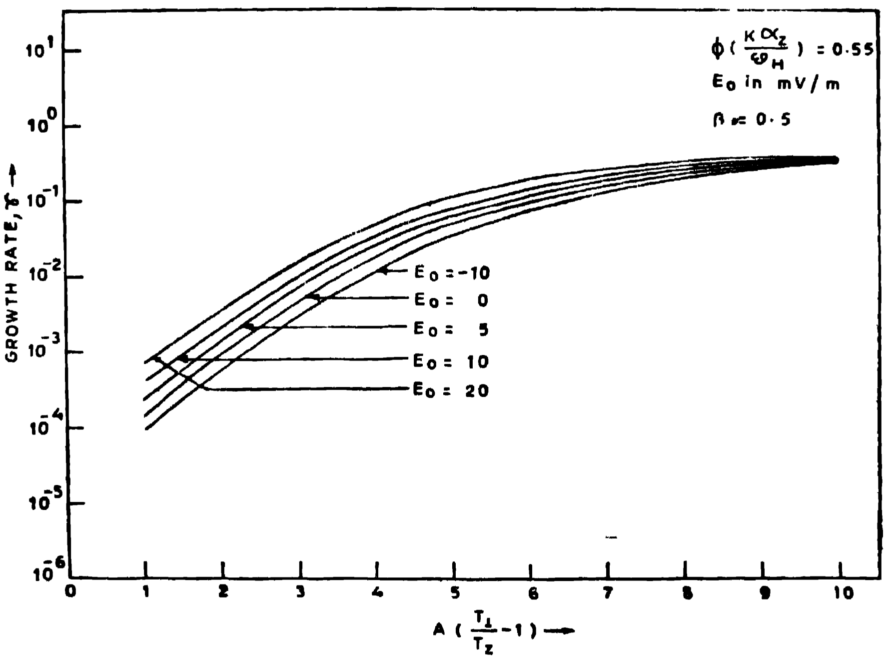

Figure 2. The variation of growth rate γ with $A\left(\frac{T_\perp}{T_\parallel}-1\right)$ for different values of E_0, $\phi\left(\frac{k\alpha_z}{\omega_H}\right)=0.55$ at $L=6.6\ R_E$.

$\phi=0.55$ has been shown in Figure 2. As the temperature anisotropy increases, the growth rate also increases and the effect of electric field minimizes when A approaches to the maximum value. In Figure 3, variation of γ vs E_0 has been plotted. Ion cyclotron electromagnetic waves propagating through the region of parallel ($\boldsymbol{k} \parallel \boldsymbol{E}_0$) or perpendicular ($\boldsymbol{k} \perp \boldsymbol{E}_0$) d.c. electrostatic field may be amplified or damped.

Cornwall *et al* (1970) first pointed out that the sudden decrease in resonant energy at the plasmapause should lead to wave growth, greatly enhanced pitch

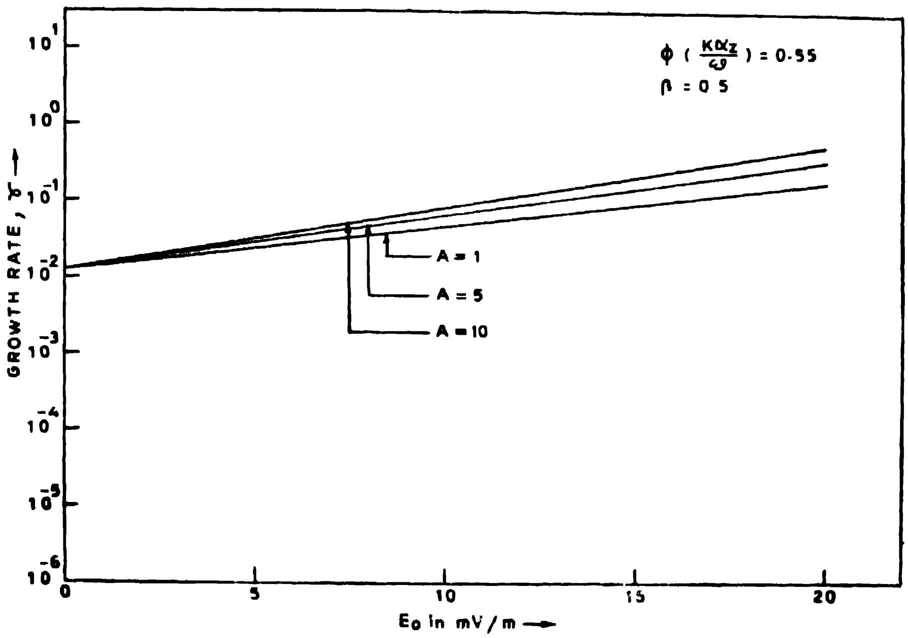

Figure 3. The variation of growth rate γ with E_0 for different values of $A\ \left(\frac{T_\perp}{T_\parallel}-1\right)$, $\phi\ \left(\frac{k\alpha_z}{\omega_H}\right)=0.55$ at $L=6.6\ R_E$.

angle scattering and rapid decay of ring currents. In subsequent reports associated theories were developed to relate the presumed ion cyclotron turbulence at the plasmapause to stable auroral red arc generation and formation of an electron slot (Coroniti 1973). These predictions are in good agreement with some energetic particle observed from Explorer 45 in the sense that during the recovery phase of geomagnetic storms, the inner edge of the proteon ring current decays just within the plasmapause in a manner consistent with expectations for a moderate ion cyclotron instability that does not yield strong diffusion.

References

Bers A and Brueck S R J 1968 *Quart. Prog. Rep.* (Res. Lab. Elect. Mass. Inst. Technol., Cambridge) **89** 156

Cornwall J M, Coroniti F V and Thorne R M 1970 *J. Geophys. Res.* **75** 4699
Cornwall J M and Schulz M 1971 *J. Geophys. Res.* **76** 7791
Coroniti F V 1973 *Radio Sci.* **8** 1007
Cuperman S, Comberoff L and Stern Lieb A 1975 *J. Plasma Phys.* **13** 259
Hsieh H C 1967 *J. Atmos. Terr. Phys.* **29** 1219
— 1968a *Phys. Fluids* **11** 1497
— 1968b *J. Atmos. Terr. Phys.* **30** 1219
Kennel C F and Petschek H E 1966 *J. Geophys. Res.* **71** 1
McCormac B M 1971 *Earth's Magnetospheric Processes* (Dordrecht, Netherlands: D Reidel)
Pines D and Schrieffer J R 1961 *Phys. Rev.* **124** 1387
Williams D J 1970 *Sources, Losses and Transport of Magneto-Spherically Trapped Particles in Solar Terrestrial Physics* Vol 3 ed E R Dyer 66